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Timelike T-duality, de Sitter space, large N gauge theories and topological field theory

C.M. Hull

Physics Department, Queen Mary and Westfield College
Mile End Road, London E1 4NS, U.K.
E-mail: C.M.Hull@qmw.ac.uk

Abstract: T-Duality on a timelike circle does not interchange IIA and IIB string theories, but takes the IIA theory to a type IIB* theory and the IIB theory to a type IIA* theory. The type II* theories admit E-branes, which are the images of the type II D-branes under timelike T-duality and correspond to imposing Dirichlet boundary conditions in time as well as some of the spatial directions. The effective action describing an En-brane is the n-dimensional Euclidean super-Yang-Mills theory obtained by dimensionally reducing 9+1 dimensional super-Yang-Mills on $9-n$ spatial dimensions and one time dimension. The IIB* theory has a solution which is the product of 5-dimensional de Sitter space and a 5-hyperboloid, and the E4-brane corresponds to a non-singular complete solution which interpolates between this solution and flat space. This leads to a duality between the large N limit of the Euclidean 4-dimensional $U(N)$ super-Yang-Mills theory and the IIB* string theory in de Sitter space, and both are invariant under the same de Sitter supergroup. This theory can be twisted to obtain a large N topological gauge theory and its topological string theory dual. Flat space-time appears to be an unstable vacuum for the type II* theories, but they have supersymmetric cosmological solutions.

Keywords: D-branes, M-Theory, String Duality, Superstrings and Heterotic Strings.

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1. Introduction

In [1], bosonic and heterotic strings were considered in toroidal backgrounds that included a compact time coordinate, and the corresponding T-duality symmetries were investigated. Although such backgrounds can have pathological features associated with closed timelike loops, they can be useful in studying the symmetries of string theory. The more dimensions that are compactified, the larger the duality symmetry [2]-[5], and a background in which all space and time dimensions are compactified would give a phase with a huge duality symmetry [3]-[5]. Moreover, there are clearly solutions of string theory which have compactified time, and studying these can give

important information about string theory. Branes wrapped both around compact time and compact space dimensions have recently been considered in [6].

Compactification of the bosonic string on a spatial n torus T^n gives a theory with a moduli space

$$\frac{O(n,n)}{O(n)\times O(n)}\times\mathbb{R}^+ \tag{1.1}$$

and a T-duality group $O(n,n;\mathbb{Z})$ while compactification on a Lorentzian torus $T^{n-1,1}$ with $n-1$ spacelike circles and one timelike one gives a moduli space

$$\frac{O(n,n)}{O(n-1,1)\times O(n-1,1)}\times\mathbb{R}^+ \tag{1.2}$$

and the T-duality group remains $O(n,n;\mathbb{Z})$. T-duality in a timelike direction swaps the time component P^0 of the momentum with the number of times a string wraps around the timelike circle and so has some unusual features. Nevertheless, the arguments of [7]-[11] show that it should be a perturbative symmetry for bosonic or heterotic strings in a background with a timelike Killing vector with compact orbits, as well as for spacelike ones.

In [12], and in [13, 14], the compactification of type II theories on Lorentzian tori $T^{n-1,1}$ was considered, and it was found that the U-duality group remained the same as that for compactification on a spacelike torus T^n, but the moduli space, which was a coset space G/H for the T^n reduction, became $G/\tilde{H}$ where $\tilde{H}$ is a certain non-compact form of H. One might expect that timelike T-duality in type II theories should give a straightforward generalisation of that for the bosonic and heterotic strings but, as we shall see, this is not the case and there are a number of surprises. This is the first of a series of papers investigating timelike T-duality and its consequences in type II string theory and in M-theory; one of the motivations is to further investigate the various limits of M-theory and the web of dualities linking them.

It will be argued that the timelike T-duals of the IIA and IIB string theories are new theories that will be referred to as the IIB* and IIA* string theories, respectively. It should be stressed that these theories can be written down directly in 9+1 non-compact dimensions; timelike compactifications are only used to link these with the type II theories, so that the IIA (IIB) theory on a timelike circle of radius R is equivalent to the IIB* (IIA*) theory on a timelike circle of radius $1/R$. If, for some reason, the type II string theories could not be formulated in a toroidal background with a timelike circle, then the type II and the type II* theories would be distinct. As will be seen, the type II* theories have a number of problems, and only some of these will be addressed in this paper. However, if timelike compactifications of type II strings are consistent, then the type II* theories should be consistent also, at least when compactified on a timelike circle.

For compactification of type II theories on a spacelike circle, T-duality identifies type IIA on a spacelike circle of radius R with type IIB on a circle of radius $1/R$ [15, 16],

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but the IIB theory is *not* the timelike T-dual of the IIA theory [13]. Consider type IIA compactified on a timelike circle of radius R, which gives a theory in 9 Euclidean dimensions. The limit in which $R \to 0$ should give a decompactification to a dual string theory in 9+1 dimensions, as in the bosonic case, and the IIA theory with radius R should be equivalent to the dual theory on a timelike circle of radius $1/R$. The limit $R \to 0$ corresponds to taking M-theory on a Lorentzian torus $T^{1,1}$ in the limit in which both radii shrink to zero size. The moduli space of $T^{1,1}$ is

$$\frac{SL(2,\mathbb{R})}{SO(1,1)} \times \mathbb{R}^+ \tag{1.3}$$

and the limit should give a theory in 9+1 dimensions with scalars taking values in

$$\frac{SL(2,\mathbb{R})}{SO(1,1)} \tag{1.4}$$

This is different from the usual IIB coset space

$$\frac{SL(2,\mathbb{R})}{SO(2)} \tag{1.5}$$

and so the theory cannot be the usual IIB theory. We will refer to the timelike T-dual of the IIA theory as the type IIB* theory and to the timelike T-dual of the IIB theory as the type IIA* theory.

The IIB* theory has scalar coset space (1.4), and the RR scalar has a kinetic term of the wrong sign. In fact, the bosonic sectors of the IIA* and IIB* supergravities are, as will be seen in section 4, obtained from those of the IIA and IIB theories by continuing all the RR n-form gauge fields $C_n \to -iC_n$, so that the kinetic terms of all the RR gauge fields have the wrong sign. The full type IIA* and IIB* string theories have a twisted form of $N = 2$ supersymmetry and are obtained by acting on the type IIA and type IIB string theories with i^{F_L}, where F_L is the left-handed fermion number. The NS-NS sector of the supergravity is unchanged by this duality, but the RR gauge fields all become ghost-like, and the resulting supergravity theories have ghosts. In [17], similar actions were considered in which kinetic terms of the wrong sign signalled instabilities. However, when type II* string theory is compactified on a timelike circle, it is equivalent to a conventional type II theory on the dual timelike circle, and so it is no more pathological than the timelike compactification of this conventional theory.

Ghosts occur generically for timelike compactification of gauge theories. For example, Yang-Mills theory compactified on time gives a Euclidean theory with a scalar from the reduction of A_0 with a kinetic term of the wrong sign, but in the full higher dimensional theory, A_0 can be gauged away, at least in the topologically trivial sector without Wilson lines in the timelike direction. Thus if all the Kaluza-Klein modes are kept, the theory should be ghost-free (in the topologically trivial sector) as a result

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of the higher-dimensional gauge invariance, but if the Kaluza-Klein modes are truncated the resulting theory has a scalar ghost. It is sometimes possible to twist such a Euclidean-space gauge theory obtained via a timelike reduction to obtain a well-defined topological theory [18, 19]. These matters will be discussed further later. However, the situation for the type II* theories might be similar. If the type II* string theories are truncated down to their supergravity limits, the supergravity theories have ghosts. However, in the full string theories, it is possible that the string gauge symmetries can be used to eliminate the ghosts. Indeed, the type II* theories are linked by T-duality to the type II theories which are ghost-free, at least perturbatively.

It is possible that, for some reason, closed timelike curves are not allowed in string theory or M-theory, and there is a stringy 'chronology protection principle' analogous to that proposed in [20], in which case the theories obtained by postulating a compact time and performing a timelike T-duality might make the inconsistencies or instabilities more apparent. Even in that case, there would be topological twisted versions of these theories that should be consistent, as we will see. However, it is clear that if timelike compactifications of type II theories are contemplated, one is led ineluctably to consider type II* theories.

The type II* string theories exist in their own right in $9+1$ dimensional Minkowski space, irrespective of whether or not closed timelike curves are 'allowed'. However, the link to the type II theories through timelike T-duality guarantees that much of the formal structure of the type II theories immediately carries over to the type II* theories, which might be thought of as a different real form of an underlying complexified theory. In particular, the type II* theories will be supersymmetric and many of their properties can be found by simply tracing the sign changes that come about in going from the type II to the type II* theories.

Type II theories have Dp-branes corresponding to imposing Dirichlet boundary conditions in $9-p$ spatial dimensions $x^{p+1}, ..., x^{9-p}$ and Neumann conditions in the remaining dimensions $t, x^1, ..., x^p$, so that the ends of the string are confined to a $p+1$ dimensional timelike Dp-brane parameterised by $t, x^1, ..., x^p$ [15]. A T-duality in the time direction changes the boundary conditions in t from Neumann to Dirichlet, so that the end of the string is confined to a spacelike surface at fixed $t, x^{p+1}, ..., x^{9-p}$ parameterised by $x^1, ..., x^p$. We will refer to this p-dimensional spacelike surface as an Ep-brane; for example, an E0-brane is at a fixed point or event in space and time and so is a Minkowski space analogue of a D-instanton [21, 22]. An E1-brane can be thought of as the spacelike world-line of a tachyon, so that whereas a D0-brane is associated with a particle, an E1-brane is associated with a tachyon. The En-branes with $n > 1$ can be thought of as higher dimensional analogues of a tachyon and the corresponding supergravity solutions might be thought of as the field configuration due to such tachyons. As tachyons are associated with vacuum instability, it might be expected that the E-branes might signal some instability; this will be addressed in section 11.

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Note, however, that gauged supergravities have tachyons (particles with negative mass squared) but that nonetheless anti-de Sitter space solutions can be completely stable and the space-time curvature tames the potential instabilites [25].

The type II* theories do not have D-branes, but have E-branes instead. We will be careful to distinguish between Ep-branes, which occur in the Lorentzian signature type II* theories, and D-instantons (and their D$(p+1)$-instanton generalisations [21]), which occur in the Wick rotated theory.

The world-volume theory of an Ep-brane is the p-dimensional Euclidean super Yang-Mills theory obtained by compactifying the usual 9+1 dimensional super Yang-Mills theory on $T^{9-p,1}$. In particular, N coincident E4-branes gives a superconformal theory in 4 Euclidean dimensions with $U(N)$ gauge symmetry, and arguments similar to those of Maldacena [26] lead, as will be shown in sections 8, 9 and 10, to the conjecture that the large N limit should be equivalent to type IIB* string theory in a 5 dimensional de Sitter space background (so that the 4-dimensional Euclidean conformal group $SO(5,1)$ becomes the 5-dimensional de Sitter group). Moreover, the Euclidean super Yang-Mills theory can be twisted to obtain a topological field theory [18, 19], so that there should be a corresponding twist of the type IIB* string theory to obtain a topological string theory, which is dual to the large N limit of the $U(N)$ topological super Yang-Mills theory. The physical interpretation of E-branes will be considered in section 11.

2. Euclidean superconformal symmetry and super Yang-Mills theory

The field content of super Yang-Mills in 9+1 dimensions is a vector field A_M and a Majorana-Weyl spinor λ, both of which are Lie-algebra-valued. Dimensional reduction on a spacelike torus T^n gives a theory in $10-n$ dimensions with a vector A_μ and n scalars ϕ^i from the internal components A_i of A_M. There is an $SO(n)$ R-symmetry arising from the original 10-dimensional Lorentz symmetry under which the scalars transform as a vector and the fermion fields transform according to a spinor representation.

Euclidean Yang-Mills theory in ten dimensions can similarly be reduced on T^n to give a vector and n scalars with an $SO(n)$ symmetry and this can be viewed as the Wick rotation of the bosonic sector of the corresponding super Yang-Mills theory. However, the Wick rotation of the fermion sectors is problematic; for example, there is no Majorana-Weyl spinor in ten Euclidean dimensions. If a Weyl or Majorana spinor is used, then the number of fermion fields is doubled and there is no supersymmetry. One approach to Euclidean quantization is that of Osterwalder and Schrader [27]. Dimensionally reducing the fermion-doubled theory again gives a fermion-doubled theory in $10-n$ dimensions with the fermions transforming as a spinor of $SO(n)$. There are a number of ways of dealing with this in the quantum theory and the Euclidean functional integral – see [28] for a recent discussion and list of references – but there

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is usually no supersymmetric Euclidean action; for example, the Wick rotation of the bosonic sector of the super Yang-Mills theory has no supersymmetric completion.

Alternatively, one can reduce the 9+1 dimensional super Yang-Mills theory on one time and $n-1$ spatial dimensions to obtain a theory in $10-n$ Euclidean dimensions. This has 16 supersymmetries and an $SO(n-1,1)$ R-symmetry under which the scalars transform as a vector and the fermions as a spinor. The scalar from the time component A_0 of the vector potential has a kinetic term of the wrong sign, so that the theory has ghosts.

Thus in $D = 10-n$ Lorentzian dimensions there is the usual super Yang-Mills theory with 16 supersymmetries and $SO(n)$ R-symmetry (or more properly, the double cover $Spin(n)$), and any Wick rotation of this would be a theory in D Euclidean dimensions, again with $SO(n)$ R-symmetry. However, the fermion sector and supersymmetry are problematic. There is also a Euclidean theory in D dimensions obtained by timelike reduction, with $SO(n-1,1)$ R-symmetry and 16 supersymmetries. We will refer to the Wick-rotated theory as 'Euclideanised' and the timelike reduction as 'Euclidean'.

For example, in $D = 4$, the Lorentzian $N = 4$ theory has $SU(4) \sim SO(6)$ R-symmetry, the Euclideanised theory again has an $SU(4)$ internal symmetry, while the Euclidean theory has $SO(5,1)$ R-symmetry and is invariant under an $N = 4$ Euclidean super-Poincaré algebra under which the 4 supercharges transform as a **4** of $SO(5,1)$. The spinorial generators arise as $SU(2)$-pseudo-Majorana-Weyl spinors of $SO(5,1)$, in the terminology of [29].

The Euclidean theory arising from a timelike reduction and a truncation of the Kaluza-Klein modes gives a non-unitary theory, but if the full Kaluza-Klein spectrum is kept, the theory is the original unitary gauge theory, albeit on a background that includes a timelike torus, $T^{n-1,1}$. In particular, one can choose a physical gauge in which the longitudinal component A_0 is eliminated locally, so that the scalar with a kinetic term of the wrong sign is removed (locally). The full theory is ghost-free (at least in the sector without timelike Wilson lines) and it is the truncation of the Kaluza-Klein modes that leads to a theory with ghosts.

There are three possible twistings of $N = 4$ super Yang-Mills to give topological field theories [30, 31]. The nature of these twistings has recently been clarified in [18, 19], where it was shown that these theories are best understood as twistings of the $D = 4$ Euclidean super Yang-Mills theory with $SO(5,1)$ R-symmetry and $SO(4) \sim SU(2)_L \times SU(2)_R$ Lorentz symmetry [18, 19]. The three distinct topological theories arise from twisting the Lorentz symmetry with a subgroup of the R-symmetry. Twisting the $SO(4)$ Lorentz symmetry with $SO(4) \subset SO(5,1)$ gives the B-model with an $SO(1,1)$ internal symmetry, twisting the $SU(2)_L$ subgroup of the Lorentz symmetry with the subgroup $SU(2)_A$ of the R-symmetry, with the embedding $SU(2)_A \times SU(2)_B \times SO(1,1) \subset SO(5,1)$, gives the half twisted model with $SU(2) \times SO(1,1)$ internal symmetry, while twisting the $SU(2)_L$ subgroup of the Lorentz symmetry with the subgroup

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$SO(3)$ of the R-symmetry, with the embedding $SO(3) \times SO(2,1) \subset SO(5,1)$, gives the A-model with $SO(2,1)$ internal symmetry. The actions for all of these topological theories contain fields with kinetic terms of the wrong sign.

For the super Yang-Mills theory in D dimensions on a D-manifold M, there is a quasi-topological theory if M has special holonomy so that it admits commuting Killing spinors α, which can be used to construct a nilpotent BRST operator, so that the physical states are defined to be BRST cohomology classes. These give higher dimensional analogues of topological field theories [18, 19, 32].

The usual super Yang-Mills theory in 2+1 Lorentzian dimensions has an $SO(7)$ R-symmetry and dualising the vector to an extra scalar gives a theory with 8 scalars and an $SO(8)$ R-symmetry, which is the M2-brane world-volume theory. This has a conformal fixed point at which the Poincaré symmetry is enlarged to the 3-dimensional conformal group $SO(3,2)$ [33]. The full supersymmetric theory is superconformally invariant, invariant under the superconformal group $OSp(4/8)$, whose bosonic subgroup is $SO(3,2) \times SO(8)$. The Euclideanised version of this should have $SO(4,1) \times SO(8)$ symmetry (with $SO(4,1)$ the conformal group in 3 Euclidean dimensions) but, as was to be expected, there is no superconformal algebra containing this. The Euclidean super Yang-Mills theory obtained by reduction on $T^{6,1}$, however, has 7 scalars, one vector and an $SO(6,1)$ R-symmetry. Dualising the vector gives an extra scalar (with kinetic term of the wrong sign). This results in a theory with 16 supersymmetries and $SO(6,2) = SO^*(8)$ R-symmetry. There are 8 scalars transforming as a vector of $SO(6,2)$, and two of the scalars are ghosts. This theory should also have a conformal point at which it is invariant under $SO(4,1) \times SO(6,2)$. It is also supersymmetric and so is in fact superconformally invariant, with a superconformal group whose bosonic subgroup is $SO(4,1) \times SO(6,2)$ and which was found in [34], where it was named $OSp^*(4/8)$, as it is a different real form of $OSp(4/8)$. (There is no superconformal group whose bosonic subgroup is $SO(4,1) \times SO(7,1)$ [34], confirming the sign of the extra scalar's kinetic term.)

The $N = 4$ super Yang-Mills theory in 3+1 Lorentzian dimensions is superconformally invariant, and the superconformal group is $SU(2,2/4)$, whose bosonic subgroup is the product of the $SO(6) \sim SU(4)$ R-symmetry and the $SO(4,2) \sim SU(2,2)$ conformal group in 3+1 dimensions. The Euclideanised version (from Wick rotation) has $SO(5,1) \times SO(6)$ symmetry (with $SO(5,1)$ the conformal group in 4 Euclidean dimensions), and played an important role in [36]; however, there is no Euclidean superconformal group in this case. The Euclidean version (from timelike dimensional reduction), however, is conformally invariant and supersymmetric and so must be invariant under a superconformal symmetry whose bosonic subgroup is $SO(5,1) \times SO(5,1)$ and which contains the super-Poincaré algebra. It is straightforward to derive it as the symmetry algebra of the Euclidean super Yang-Mills, and is a different real form of $SU(2,2/4)$, which will be denoted $SU^*(4/4)$; it was first given in [35].

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3. Euclidean D-branes and super Yang-Mills

In type II string theory one can have open strings that satisfy Neumann boundary conditions in $p+1$ dimensions X^μ, $\mu = 0, 1, ..., p$, and Dirichlet boundary conditions in the remaining $9-p$ dimensions X^i, $i = 1, ..., 9-p$. (Here $0 \leq p \leq 9$ and p is odd for type IIB and even for type IIA theories). These constrain the ends of the strings to lie in a $p+1$ surface $X^i = constant$, which is a Dp-brane [15]. These are dynamical extended 'solitons'. T-duality in a transverse direction X^{i_1} (assumed to be compact) changes the X^{i_1} boundary condition from Dirichlet to Neumann, taking the Dp-brane to a D$p+1$ brane, and conversely a T-duality in a spatial longitudinal direction takes a Dp-brane to a D$p-1$ brane.

In the Wick-rotated or Euclideanised string theory, Dirichlet boundary conditions in $10-q$ (with $0 \leq q \leq 10$) dimensions gives a q-dimensional Euclidean surface which is interpreted as an extended instanton [21]. These Euclideanised D-branes are sometimes referred to as q-instantons or (confusingly) as D$q-1$ branes, so that an instanton which is pointlike in 10 Euclidean dimensions is a p-brane with $p = -1$. These instantonic branes give important contributions to the Euclidean functional integral [23, 24].

However, it is also consistent (formally) to consider boundary conditions for string theory with Lorentzian signature in which Dirichlet conditions are imposed in the time direction. Then open strings satisfy Neumann boundary conditions in p dimensions X^μ, $\mu = 1, ..., p$, and Dirichlet boundary conditions in the remaining $10-p$ dimensions X^0, X^i, $i = 1, ..., 9-p$, giving a spacelike p-dimensional surface $X^i = constant$, $X^0 = constant$ at a fixed instant in time. These Euclidean p-branes will be referred to here as E-branes, so as to distinguish them from D-branes. An E0-brane, for example, is located at a point in 10-dimensional space-time. Although the interpretation of such branes is unclear, they occur in the theory and one of the aims of this paper will be to study some of their properties.

If the time dimension is compact, a T-duality in the time direction takes a Dp-brane to an Ep-brane, and an Ep-brane to a Dp-brane. Thus if T-duality in a compact time-direction is allowed, as is usually supposed, then there must be Ep-branes in the theory. T-duality in a longitudinal direction takes an Ep-brane to an E$(p-1)$-brane while T-duality in a spacelike transverse direction takes an Ep-brane to an E$(p+1)$-brane. Ep-branes with p even occur in the IIB* theory obtained from type IIA via a T-duality on a timelike circle, while Ep-branes with p odd occur in the IIA* theory obtained from type IIB from a timelike T-duality.

The dynamics of a D-brane is given in terms of the strings ending on it, so that the low-energy effective dynamics are given by the zero-slope limit of these strings, which is $p+1$ dimensional super Yang-Mills on the brane. In particular, the scalar fields are collective coordinates for the brane position. Similarly, the effective description of D-branes in Wick-rotated string theory is Euclideanised or Wick-rotated super Yang-

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Mills, while that of an Ep-brane is the Euclidean super Yang-Mills theory obtained by reducing from $9+1$ dimensions in $9-p$ spatial and one time dimensions. The ghost scalar field is the collective coordinate for the time direction.

4. Timelike T-duality and type II theories

IIA string theory compactified on a spacelike circle of radius R is T-dual to IIB string theory compactified on a circle of radius $1/R$ [15, 16]; the two type II theories are different decompactification limits of the same theory. The low-energy limits of the two string theories are the type IIA and type IIB supergravity theories and the T-duality is reflected in the fact that the dimensional reduction of the two supergravities to 9 dimensions give the same 9-dimensional supergravity theory [37]. However, whereas for supergravity compactified on a circle of radius R, the limit $R \to 0$ gives a truncation to nine-dimensional supergravity, the corresponding limit for string theory is a decompactification to the T-dual string theory.

Consider now the case of timelike reductions [12, 13] and T-duality. For the bosonic and heterotic strings, timelike T-duality is straightforward (at least perturbatively), and takes the theory on a circle to the same theory on a dual circle [1]. This is straightforward to check by manipulations of the functional integral, and the transformations of the background metric, dilaton and 2-form fields are those of [7, 8] (with the shift of the dilaton proportional to $\log|k^2|$, where k^2 is the length squared of the Killing vector in the direction being dualised). This T-duality is reflected in the $U(1)$ symmetry of the dimensionally reduced 9-dimensional classical field theory under which the Maxwell fields from the metric and the 2-form field are rotated into one another. In the quantum theory, this is broken to a discrete $\mathbb{Z}_2$ subgroup, the T-duality.

For the type II theory, the situation is more complicated. The timelike reduction of the type IIA and type IIB supergravities gives two *different* 9-dimensional Euclidean supergravities [13]. Indeed, type IIB on a timelike S^1 has scalars taking values in $SL(2,\mathbb{R})/SO(2)\times\mathbb{R}^+$, where the modulus in $\mathbb{R}^+$ is the radius of the compactifying circle, while IIA on a timelike S^1 is the same as 11-dimensional supergravity compactified on a Lorentzian torus $T^{1,1}$ and so will have scalars taking values in $SL(2,\mathbb{R})/SO(1,1)\times\mathbb{R}^+$, the moduli space of $T^{1,1}$.

However, the type IIA theory on a timelike circle of radius R should be T-dual to some string theory on a timelike circle of radius $1/R$, so that the limit $R\to 0$ should give a T-dual string theory in 9+1 non-compact dimensions. The timelike T-duality cannot then take the IIA on a timelike circle to IIB on the dual timelike circle [13], as we will see below. Here we will assume that timelike T-duality does make sense in type II theories and see where this leads.

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The bosonic action of the IIA supergravity is

$$S_{IIA} = \int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2\right) - G_2^2 - G_4^2\right] + \\ + \frac{4}{\sqrt{3}}\int G_4 \wedge G_4 \wedge B_2 + \cdots, \tag{4.1}$$

while that of IIB supergravity is

$$S_{IIB} = \int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2\right)\right. \\ \left. -G_1^2 - G_3^2 - G_5^2\right] + \cdots \tag{4.2}$$

Here Φ is the dilaton, $H = dB_2$ is the field strength of the NS-NS 2-form gauge field B_2 and $G_{n+1} = dC_n + \cdots$ is the field strength for the RR n-form gauge field C_n. The field equations derived from the IIB action (4.2) are supplemented with the self-duality constraint $G_5 = *G_5$. The dimensional reduction of either on a spacelike circle gives the type II supergravity in 8+1 dimensions, which we will refer to as the II_{8+1} theory, with bosonic action [37]

$$S_{II_{8+1}} = \int d^9x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2 - d\chi^2 - F_g^2 - F_B^2\right)\right. \\ \left. -G_1^2 - G_2^2 - G_3^2 - G_4^2\right] + \cdots, \tag{4.3}$$

where F_g, F_B are the 2-form field strengths for the NS vectors resulting from the reduction of the metric $g_{\mu\nu}$ and the NS-NS 2-form B_2 respectively, and χ is the scalar arising from the reduction of the metric. The three scalars Φ, χ, C_0 take values in the coset

$$\frac{SL(2,\mathbb{R})}{SO(2)} \times \mathbb{R}^+ \tag{4.4}$$

and the action is invariant under an $SL(2,\mathbb{R}) \times \mathbb{R}^+$ duality symmetry, broken to $SL(2,\mathbb{Z})$ in the quantum theory [5].

Consider instead the dimensional reduction on a timelike circle, to give a theory in 9 Euclidean dimensions. The reduction is similar to the usual case, but the signs of some of the kinetic terms are reversed. Timelike reduction of the metric gives a graviphoton with a kinetic term with a reversed sign, together with the usual metric and scalar, while the reduction of an n-form gauge field gives an n-form with an unchanged sign and an $n-1$ form gauge field with a kinetic term with a reversed sign. Reducing the type IIB theory gives a theory with bosonic kinetic terms

$$S_{IIB_9} = \int d^9x\sqrt{g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2 - d\chi^2 + F_g^2 + F_B^2\right)\right. \\ \left. -G_1^2 + G_2^2 - G_3^2 + G_4^2\right] + \cdots \tag{4.5}$$

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and scalars taking values in (4.4), while the timelike reduction of the IIA theory gives

$$S_{IIA_9} = \int d^9x\sqrt{g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2 - d\chi^2 + F_g^2 - F_B^2\right)\right.$$
$$\left. + G_1^2 - G_2^2 + G_3^2 - G_4^2\right] + \cdots \tag{4.6}$$

with scalars taking values in the coset space

$$\frac{SL(2,\mathbb{R})}{SO(1,1)} \times \mathbb{R}^+ . \tag{4.7}$$

The RR scalar C_0 has a kinetic term of the wrong sign, resulting in a coset space $SL(2,\mathbb{R})/SO(1,1) \times \mathbb{R}^+$ with a Lorentzian metric. We shall refer to these as the type IIB_9 and type IIA_9 theories, respectively; they differ from each other in the signs of some of the kinetic terms.

The type IIA_9 theory cannot be obtained by reduction of the IIB theory, and the type IIB_9 theory cannot be obtained by reduction of the IIA theory, so that the IIA and IIB theories cannot be related by a T-duality on a timelike circle. However, the type II theories should have timelike T-duals and these should have effective supergravity actions that dimensionally reduce to the IIA_9 and IIB_9 actions. In the NS-NS sector, the timelike T-duality acts straightforwardly through Buscher-type transformations, but there must be some sign changes in the RR sector. Consider the type IIA* and type IIB* actions given by reversing the signs of the RR kinetic terms in (4.1), (4.2) to give

$$S_{IIA^*} = \int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2\right)\right.$$
$$\left. + G_2^2 + G_4^2\right] + \cdots \tag{4.8}$$

and

$$S_{IIB^*} = \int d^{10}x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2\right)\right.$$
$$\left. + G_1^2 + G_3^2 + G_5^2\right] + \cdots , \tag{4.9}$$

where the field equations from (4.9) supplemented by the constraint $G_5 = *G_5$. The timelike reduction of the type IIA* action (4.8) gives the IIB_9 action (4.5) and that of the type IIB* action (4.9) gives the IIA_9 action (4.6). This suggests that timelike T-duality relates IIA to a IIB* theory with effective bosonic supergravity action (4.9), and the IIB to a IIA* theory with effective bosonic supergravity action (4.8). It is not hard to see that there are such IIA* and IIB* supergravity theories. The two supergravity theories result from coupling $N = 1$ supergravity to the two types of $N = 1$ gravitino multiplets, but choosing the wrong sign for the kinetic terms of the gravitino multiplets, and the resulting theory can be obtained from the usual theory by multiplying the fields

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in the gravitino multiplet by i. This can be extended to the full string theories, with the IIA* (IIB*) theory obtained by acting on the IIA (IIB) theory with i^{F_L} where F_L is the left-handed fermion number operator.

The scalars Φ, C_0 of the IIB* theory take values in the coset space

$$\frac{SL(2,\mathbb{R})}{SO(1,1)} \tag{4.10}$$

and the global $N = 2$ superalgebra for either the IIA* or IIB* theory is the twisted one

$$\{Q_i, Q_j\} = \gamma^\mu P_\mu \eta_{ij} \,, \tag{4.11}$$

where $i, j = 1, 2$ labels the two supercharges (which have the same chirality in the IIB* theory and opposite chirality in the IIA* theory) and η_{ij} is the $SO(1,1)$ invariant metric $diag(1, -1)$. The anti-commutator of the second supercharge with itself has the 'wrong' sign. (Similar twisted superalgebras in two dimensions were considered in [38].)

Thus we have been led to the construction of type IIA* and IIB* string theories related by timelike T-duality to the usual IIB and IIA string theories. The truncation to the corresponding type IIA* and IIB* supergravity theories gives theories with ghosts, but the full type II* string theories, at least when compactified on a timelike circle, are equivalent to type II string theories on the dual timelike circle. The uncompactified type II theories are ghost-free, and a physical gauge can be chosen in which longitudinal oscillations are eliminated. If the type II theories remain ghost-free when compactified on a timelike circle, then the type II* theories would be ghost-free also, at least when compactified on a timelike circle. The situation would then be similar to that encountered in the case of Yang-Mills in section 2, in which timelike dimensional reduction and truncation led to a theory with ghosts, but if the full Kaluza-Klein spectrum was kept, then the theory was ghost-free. Backgrounds with a timelike circle appear to be consistent string backgrounds, and it is interesting to understand the properties of strings in such backgrounds, and in particular the questions of stability and unitarity. It is intriguing that the problems appear to be arising only in the RR sector and not in the NS-NS sector.

5. Dimensional reduction

The dimensional reduction of the type IIA* and type IIB* supergravity theories on a spacelike circle give the same theory in 8+1 dimensions, which we will refer to as the II^*_{8+1} theory. The bosonic kinetic terms are obtained from those of (4.3) by reversing the signs of the RR kinetic terms to obtain

$$\begin{aligned} S_{II^*_{8+1}} = \int d^9x\sqrt{-g}\,\Big[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - H^2 - d\chi^2 - F_g^2 - F_B^2\right) \\ + G_1^2 + G_2^2 + G_3^2 + G_4^2\Big] + \cdots . \end{aligned} \tag{5.1}$$

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D=11-d	G_d	H_d	U-Duality
9	$SL(2,\mathbb{R}) \times SO(1,1)$	$SO(2)$	$SL(2,\mathbb{Z})$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	$SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$
7	$SL(5,\mathbb{R})$	$SO(5)$	$SL(5,\mathbb{Z})$
6	$SO(5,5)$	$SO(5) \times SO(5)$	$SO(5,5;\mathbb{Z})$
5	$E_{6(6)}$	$USp(8)$	$E_{6(6)}(\mathbb{Z})$
4	$E_{7(7)}$	$SU(8)$	$E_{7(7)}(\mathbb{Z})$
3	$E_{8(8)}$	$SO(16)$	$E_{8(8)}(\mathbb{Z})$

Table 1: Duality symmetries for 11 dimensional supergravity reduced, and M-theory compactified to $D = 11 - d$ dimensions on T^d. The classical scalar symmetric space is G_d/H_d and the U-duality group is $G_d(\mathbb{Z})$.

In fact, the type IIA* and IIB* string theories are related by T-duality on a spacelike circle, so that the type IIB* theory on a spacelike circle of radius R is equivalent to the type IIA* theory on a spacelike circle of radius $1/R$. Indeed, consider type IIA theory on a Lorentzian torus $T^{1,1}$ with spacelike radius R and timelike radius T. The moduli space of such reductions can be represented by the square in Figure 1, and the spacelike T-duality of the IIA* and IIB* theories follows from that between IIA and IIB. Compactifying the IIA (IIB) string theory on $T^{1,1}$ and taking the limit in which the torus shrinks to zero size gives the IIA* (IIB*) string theory.

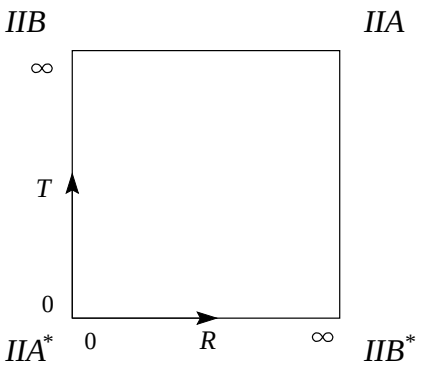

Figure 1: The moduli space for the type IIA theory compactified on a Lorentzian torus $T^{1,1}$ with spacelike radius R and timelike radius T.

The standard reduction of 11-dimensional supergravity [2, 3] on a torus T^d or of type IIA supergravity on T^{d-1} gives a supergravity theory in $11-d$ dimensions which is invariant under a rigid duality symmetry G_d and a local symmetry H_d; the groups G_d, H_d are listed in Table 1. H_d is the maximal compact subgroup of G_d and the theory has scalars taking values in the coset G_d/H_d.

The standard dimensional reduction of 11-dimensional supergravity on a Lorentzian torus $T^{d-1,1}$ or of type IIA supergravity on $T^{d-2,1}$ gives a supergravity theory in $11-d$ dimensions with G_d rigid symmetry and local $\tilde{H}_d$ symmetry for the groups $G_d, \tilde{H}_d$ listed in Table 2.

As dimensional reduction of IIA,IIB,IIA* or IIB* supergravities on $T^{1,1}$ give the same 8-dimensional Euclidean supergravity theory, the reduction of IIA* or IIB* supergravities on $T^{d-1,1}$ with $d \geq 2$ gives the Euclidean supergravity theories listed in Table 2 obtained by reducing IIA or IIB on $T^{d-1,1}$. However, the spatial reduction of

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D=11-d	G_d	H_d	$\tilde{H}_d$	K_d
9	$SL(2,\mathbb{R}) \times SO(1,1)$	$SO(2)$	$SO(1,1)$	$\mathbb{1}$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	$SO(2,1) \times SO(2)$	$SO(2) \times SO(2)$
7	$SL(5,\mathbb{R})$	$SO(5)$	$SO(3,2)$	$SO(3) \times SO(2)$
6	$SO(5,5)$	$SO(5) \times SO(5)$	$SO(5,C)$	$SO(5)$
5	$E_{6(6)}$	$USp(8)$	$USp(4,4)$	$SO(5) \times SO(5)$
4	$E_{7(7)}$	$SU(8)$	$SU^*(8)$	$USp(8)$
3	$E_{8(8)}$	$SO(16)$	$SO^*(16)$	$U(8)$

Table 2: Toroidal reductions of M-theory to $D = 11 - d$. Reduction on T^d gives the scalar coset space G_d/H_d while reduction on $T^{d-1,1}$ gives the scalar coset space $G_d/\tilde{H}_d$. $\tilde{H}_d$ is a non-compact form of H_d with maximal compact subgroup K_d.

the IIA, B^* theories gives a new class of supergravities. Reduction of IIA* or IIB* on T^n or, equivalently, the reduction of the II^*_{8+1} theory on T^{n-1} gives a supergravity theory whose scalars take values in G_d/H^*_d with $d = n+1$. The coset G_d/H^*_d is obtained by analytically continuing all the RR scalars in G_d/H_d to change them from spacelike to timelike directions. The NS-NS scalars take values in a coset space J_{n+1}/K^*_{n+1} given by

$$\frac{SO(n,n)}{SO(n) \times SO(n)} \times \mathbb{R}^+ \tag{5.2}$$

for $n \leq 5$, by

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \times \frac{SL(2,\mathbb{R})}{U(1)} \tag{5.3}$$

for $n = 6$, and

$$\frac{SO(8,8)}{SO(8) \times SO(8)} \times \mathbb{R}^+ \tag{5.4}$$

for $n = 7$. Then H^*_d is a different real form of H_d with maximal compact subgroup of K^*_{n+1}, which is $SO(n) \times SO(n)$ for $n \leq 5$. This gives the symmetries in Table 3.

These supergravities are in Lorentzian space with signature $D - 1, 1$ and have a twisted superalgebra obtained from the dimensional reduction of the twisted superalgebra (4.11). The twisted superalgebra is of the form

$$\{Q^i, Q^j\} = \eta^{ij} P \cdot \gamma \,, \tag{5.5}$$

where the supercharges are r-component spinors and the index $i = 1, \ldots, s = 32/r$ labels the supercharges and transforms under an s-dimensional representation of the automorphism group H^*_d. The metric η^{ij} (which may be symmetric or antisymetric, depending on the dimension) is invariant under H^*_d and has $s/2$ eigenvalues of $+1$ and $s/2$ eigenvalues of -1. For example, for $D = 3$, there are 16 2-component supercharges Q^i, $i = 1, \ldots, 16$, which transform as a **16** of $SO(8,8)$ and η^{ij} is the $SO(8,8)$-invariant

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D=11-d	G_d	H_d	H_d^*	K_d^*
9	$SL(2,\mathbb{R}) \times SO(1,1)$	$SO(2)$	$SO(1,1)$	$\mathbb{1}$
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	$SO(2,1) \times SO(2)$	$SO(2) \times SO(2)$
7	$SL(5,\mathbb{R})$	$SO(5)$	$SO(4,1)$	$SO(3) \times SO(3)$
6	$SO(5,5)$	$SO(5) \times SO(5)$	$SO(4,1) \times SO(4,1)$	$SO(4) \times SO(4)$
5	$E_{6(6)}$	$USp(8)$	$USp(4,4)$	$SO(5) \times SO(5)$
4	$E_{7(7)}$	$SU(8)$	$SU(4,4)$	$SO(6) \times SO(6) \times U(1)$
3	$E_{8(8)}$	$SO(16)$	$SO(8,8)$	$SO(8) \times SO(8)$

Table 3: Reductions on spacelike tori T^{d-1} of the type IIA* theory to $D = 11 - d$. The scalar coset space is G_d/H_d^*, where H_d^* is a non-compact form of H_d with maximal compact subgroup K_d^*.

metric, while for $D = 5$, $i = 1, ..., 8$ and η^{ij} is the antisymmetric matrix preserved by $USp(4,4)$. Note that $H_d^* \sim \tilde{H}_d$ for $d = 3, 6$, but even for these values of d, the theories in Tables 2 and 3 are distinct; those in Table 2 are Euclidean, while those in Table 3 are Lorentzian with twisted supersymmetry.

6. M-theory and F-theory

The IIA_9 theory was obtained by compactifying the IIA theory on a timelike circle, and so can also be obtained by compactifying M-theory on a Lorentzian $T^{1,1}$, and the scalars in the coset space (4.4)n correspond to the $T^{1,1}$ moduli. Just as the IIB theory can be obtained by compactifying M-theory on T^2 and taking the limit in which the torus shrinks to zero size [39], the IIB* theory can be obtained by taking M-theory on a $T^{1,1}$ with radii R_1, R_2 and taking the limit in which both radii tend to zero, with the IIB* coupling constant determined by the limit of the ratio R_1/R_2.

Compactifications of type IIB theory on a space K for which the complex scalar field τ is not a function on K but is the section of a bundle can be described as a compactification of F-theory on a T^2 bundle B over K [40]. If K is an n-dimensional Euclidean space, then B is an $n + 2$-dimensional Euclidean space, and the F-theory background is $11 + 1$ dimensional (i.e. eleven space and one time dimensions). On the other hand, compactifications of type IIB* theory on a space K for which the complex scalar field τ is the section of a bundle over K can be described as a compactification of an F^*-theory on a $T^{1,1}$ bundle B over K. If K is an n-dimensional Euclidean space, then B is an $n + 2$-dimensional space with Lorentzian signature, and the F^*-theory background is $10 + 2$ dimensional, with two times. 12 dimensional theories with signature $11 + 1$ were considered in [41], while the signature $10 + 2$ was considered in [40], but in the context of F-theory, not F^*-theory, and in [42].

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7. E-branes and type II* strings

The E-branes of section 3 were found from D-branes of type II string theories by a timelike T-duality, and so must occur in the type II* string theories obtained from the type II strings by a timelike T-duality. The Ep-branes for p odd occur in the type IIA* string theory and the Ep-branes for p even occur in the type IIB* string theory. The bosonic sectors of the type II* theories can be obtained from those of the type II theories by the field redefinition $C_n \to C'_n = -iC_n$ of the RR n-form potentials C_n. The Dp-brane of type II carries a real RR charge corresponding to C_{p+1}, while an Ep-brane of type II* carries a real RR charge corresponding to C'_p. An Ep-brane of type II, corresponding to having Dirichlet boundary conditions in time as well as $9-p$ spatial directions in type II string theory, would carry an *imaginary* RR C_p charge, which is then interpreted as a real C'_p charge of the type II* theory.

The type II supergravity solution for a Dp-brane, for $p < 7$, has a metric [48, 49]

$$ds^2 = H^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_p^2) + H^{1/2}(dx_{p+1}^2 + \cdots + dx_9^2)\ , \tag{7.1}$$

where H is a harmonic function of the transverse coordinates $x_{n+1}, \ldots, x_9$, and the dilaton and RR potential $C_{012\ldots p}$ are also given in terms of H. The simplest choice is

$$H = c + \frac{q}{r^{7-p}}\,, \tag{7.2}$$

where c is a constant (which can be taken to be 0 or 1), q is the D-brane charge and r is the radial coordinate defined by

$$r^2 = \sum_{i=p+1}^{9} x_i^2\,. \tag{7.3}$$

If the time coordinate t is taken to be periodic, a T-duality transformation in the t direction gives an Ep-brane solution with metric

$$ds^2 = H^{-1/2}(dx_1^2 + \cdots + dx_p^2) + H^{1/2}(-dt^2 + dx_{p+1}^2 + \cdots + dx_9^2)\,, \tag{7.4}$$

This can now be generalised to allow H to be a harmonic function of $t, x_{n+1}, \ldots, x_9$, so that it depends on time as well as the spatial transverse coordinates. In addition to the solution with H given by (7.2), there are solutions

$$H = c + \frac{q}{\tau^{8-p}} \tag{7.5}$$

and (for odd p)

$$H = c + \frac{q}{\sigma^{8-p}} \tag{7.6}$$

where τ, σ are the proper time and distance defined by

$$\tau^2 = -\sigma^2 = t^2 - r^2\,. \tag{7.7}$$

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These solutions have a potential singularity on the light-cone $t^2 = r^2$ and arise as complex solutions of the original type II supergravity, with $C_{12....p}$ imaginary, or as real solutions of the type II* theory, with $C_{12....p}$ real. Analogous solutions were considered in [17] where they were interpreted as signalling an instability. The E-branes preserve 16 of the 32 supersymmetries of the type II* theories. The E-brane solutions will be discussed in more detail in later sections and in [43].

The E0-brane is closely related to the D-instanton of [21, 22]. In [22], a 'Euclideanised' IIB theory ten Euclidean dimensions was proposed in which the RR scalar C_0 is continued to $C_0' = iC_0$, so that the action for C_0' is negative and the scalars Φ, C_0' take values in an $SL(2,/R)/SO(1,1)$ coset space. In Einstein frame, the metric of the D-instanton is just the flat 10-dimensional Euclidean metric, while the dilaton and RR scalar are given by

$$e^{\Phi} = H, \qquad C_0' = H^{-1} \tag{7.8}$$

in terms of a harmonic function $H(x_1, \ldots, x_{10})$. The spherically symmetric instanton arises from the choice

$$H = 1 + \frac{q}{r^8}\,. \tag{7.9}$$

The E0-brane solution of the type IIB* theory, which has scalars Φ, C_0 taking values in an $SL(2,\mathbb{R})/SO(1,1)$ coset space but has a Lorentzian space-time signature, is closely related. The metric is the flat 10-dimensional *Minkowski* metric, while the dilaton and RR scalar are given by

$$e^{\Phi} = H, \qquad C_0 = H^{-1}\,, \tag{7.10}$$

but now H is a harmonic function of $t, x_1, \ldots, x_9$, i.e. it satisfies the 10-dimensional wave equation; a simple choice is

$$H = 1 + \frac{q}{r^8}\,. \tag{7.11}$$

An E0-brane arises from Dirichlet conditions associated with a particular point X in space-time, and there are solutions corresponding to scalar waves emanating from the event X.

8. De Sitter space and large N gauge theory

The type IIB supergravity theory has a solution [44]

$$AdS_5 \times S^5 = \frac{SO(4,2)}{SO(4,1)} \times \frac{SO(6)}{SO(5)}\,, \tag{8.1}$$

so that compactification on a 5-sphere gives a gauged supergravity in 5-dimensional anti-de Sitter space AdS_5. The 5-dimensional anti-de Sitter space AdS_5 can be represented as the hyperboloid

$$-T^2 - t^2 + x_1^2 + \cdots + x_4^2 = a^2 \tag{8.2}$$

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in $\mathbb{R}^6$, with the metric induced from

$$ds^2 = -dT^2 - dt^2 + dx_1^2 + \cdots + dx_4^2 \,. \tag{8.3}$$

The RR 4-form gauge field has a field strength that is the self-dual combination of the volume form on AdS_5 and the volume form on S^5, and its energy-density gives rise to the negative cosmological constant. In the type IIB* theory, the RR 4-form gauge field has a kinetic term and hence an energy-momentum tensor with the 'wrong' sign, and so a similar ansatz gives a solution with a *positive* cosmological constant, and there is a de Sitter solution $dS_5 \times H^5$, where dS_5 is de Sitter space and H_5 is (one of the the two sheets of) the hyperboloid $SO(5,1)/SO(5)$, so that

$$dS_5 \times H^5 = \frac{SO(5,1)}{SO(4,1)} \times \frac{SO(5,1)}{SO(5)} \,. \tag{8.4}$$

The de Sitter space dS_5 can be represented as the hyperboloid

$$-T^2 + Y^2 + x_1^2 + \cdots + x_4^2 = a^2 \tag{8.5}$$

in $\mathbb{R}^6$, with the metric induced from the Minkowski metric

$$ds^2 = -dT^2 + dY^2 + dx_1^2 + \cdots + dx_4^2 \,, \tag{8.6}$$

while H^5 arises from the hyperboloid

$$-T^2 + Y^2 + x_1^2 + \cdots + x_4^2 = -a^2 \,, \tag{8.7}$$

again embedded in the 6-dimensional Minkowski space with metric (8.6). The hyperboloid has two sheets, and H^5 is taken to be one of the two.

Reduction on H^5 (in the sense of [46]) gives a gauging of the 5-dimensional Euclidean maximal supergravity theory from Table 3, with gauge group $SO(5,1)$ arising from the isometries of H^5. This has an $SO(5,1)$-invariant de Sitter vacuum which preserves all of the supersymmetries, i.e. it has 32 Killing spinors. The vacuum is invariant under the de Sitter supergroup $SU^*(4/4)$ of section 2, with bosonic subgroup $SO(5,1) \times SO(5,1)$. As in the de Sitter supergravity constructed in [34], some of the fields have kinetic terms of the wrong sign. Alternatively, one can reduce on dS_5 to obtain a Euclidean supergravity on H^5, which is a gauging of the 5-dimensional maximal supergravity theory from Table 2. It has $SO(5,1)$ gauge symmetry and a maximally symmetric H^5 ground state, again invariant under $SU^*(4/4)$. Although both H^5 and dS_5 are non-compact, it seems that these solutions can be given a Kaluza-Klein interpretation if suitable boundary conditions are imposed. If H^5 is regarded as the internal space, then the natural boundary conditions are the reflective boundary conditions on H^5 used in e.g. [36], giving rise to a discrete spectrum.

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Anti-de Sitter space in five dimensions is topologically $\mathbb{R}^4 \times S^1$ with the S^1 timelike, while its covering space is topologically $\mathbb{R}^5$. It has a timelike boundary, which is topologically $S^3 \times S^1$, or $S^3 \times \mathbb{R}$ in the covering space. The sphere S^3 at infinity is of infinite radius. The space H^5 was considered in detail in [36], where it arose as 'Euclidean anti-de Sitter space'. It is topologically $\mathbb{R}^5$ and has a boundary which is an S^4 at infinity. The de Sitter space dS_5 is topologically $S^4 \times \mathbb{R}$ with the $\mathbb{R}$ corresponding to time. The metric can be written as

$$ds^2 = -dt^2 + a^2 \cosh^2(t/a) d\Omega_4^2 \,, \tag{8.8}$$

where Ω_n^2 is the metric on a unit n-sphere. Thus an S^4 contracts from an infinite size at $t = -\infty$ to a minimal radius a at $t = 0$ and then re-expands to infinite size at $t = \infty$. There is a spacelike infinity which is topologically S^4 both in the future ($t = \infty$) and in the past ($t = -\infty$), and each S^4 of constant t is a Cauchy surface. The solutions of wave equations on each of these spaces are given by specifying boundary conditions at the appropriate hypersurface. In AdS_5 and H^5, there is a unique regular solution for given asymptotic boundary values, while in de Sitter space the solutions are determined by giving data on any Cauchy surface, e.g. initial data at $t = -\infty$.

Maldacena has proposed [26] that the IIB string theory on $AdS_5 \times S^5$ is dual to $\mathcal{N} = 4$ super-Yang-Mills theory with $U(N)$ gauge group, at least in the limit $N \to \infty$. Both theories have $SU(2,2/4)$ symmetry, and the conjecture was motivated by considering N coincident D3-branes. The supergravity solution given by (7.1),(7.2) with $p = 3$ interpolates between the Minkowski space and the $AdS_5 \times S^5$ vacua, and $AdS_5 \times S^5$ is the geometry which arises in the limits used in [26].

We propose here a similar duality between the type IIB* string theory on $dS_5 \times H^5$ and the Euclidean $\mathcal{N} = 4$ super-Yang-Mills theory (arising from compactifying the usual super Yang-Mills theory in 9+1 dimensions on $T^{5,1}$) with $U(N)$ gauge group, at least in the limit $N \to \infty$. Both theories have $SU^*(4/4)$ symmetry, which is interpreted as a super de Sitter group in 5 dimensions or as a superconformal group in 4 Euclidean dimensions. Note that both theories have ghosts.

This conjecture can be motivated by considering N coincident E4-branes; the corresponding type IIB* supergravity solution breaks half the supersymmetry and interpolates between the flat space solution and $dS_5 \times H^5$, both of which preserve all supersymmetries, just as the D3-brane interpolates between flat space and $AdS_5 \times S^5$ [45]. Moreover, the de Sitter solution $dS_5 \times H^5$ arises in the large N limit, as in [26].

9. Interpolating geometries and large N gauge theories

Let us begin by recalling the geometry of the D3-brane solution given by

$$ds^2 = H^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_3^2) + H^{1/2}(dr^2 + r^2 d\Omega_4^2) \,, \tag{9.1}$$

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with

$$H = 1 + \frac{a^4}{r^4}\,. \tag{9.2}$$

When $r >> a$, $H \sim 1$, so that for large r the metric approaches that of flat space. When $r << a$, the constant term in H is negligible and the metric becomes approximately

$$ds^2 = \frac{r^2}{a^2} dx_{||}^2 - \frac{a^2}{r^2} dr^2 + a^2 d\Omega_5^2\,, \tag{9.3}$$

where the longitudinal metric is $dx_{||}^2 = -dt^2 + dx_1^2 + \cdots + dx_3^2$. This metric is then that of a product of a 5-sphere of radius a and 5-dimensional anti-de Sitter space with 'anti-de Sitter radius' a and metric

$$ds^2 = \frac{r^2}{a^2} dx_{||}^2 - \frac{a^2}{r^2} dr^2\,. \tag{9.4}$$

Thus the D3 brane interpolates between flat space and the $AdS_5 \times S^5$ solution.

A similar argument shows that the E4-brane interpolates between flat space and the $dS_5 \times H^5$ solution of the IIB* theory. The $E4$-brane solution (7.4),(7.5) can be written as

$$ds^2 = H^{-1/2}(dx_1^2 + \cdots + dx_4^2) + H^{1/2}(-dt^2 + dr^2 + r^2 d\Omega_4^2)\,, \tag{9.5}$$

with

$$H = c + \frac{q}{\tau^4}\,, \tag{9.6}$$

where τ is the proper time $\tau^2 = t^2 - r^2$. Taking $c = 1$, the metric approaches the flat metric far away from the light-cone $r^2 = t^2$ when τ^4 is very large, while near the light-cone the constant term in H is negligible and, as we will see, the metric approaches that of the $dS_5 \times H^5$ solution. This is non-singular, and the E4-brane solution is itself non-singular, as we shall see. The light-cone $r^2 = t^2$ divides the space into two regions, and we will consider each separately.

Consider first the region $t^2 > r^2$. Then taking

$$H^{1/2} = \frac{a^2}{\tau^2}\,, \tag{9.7}$$

with $a^4 = q$ and defining the Rindler-type coordinates τ, β by

$$t = \tau \cosh\beta, \qquad r = \tau \sinh\beta\,, \tag{9.8}$$

the metric (9.5) becomes

$$ds^2 = \frac{\tau^2}{a^2} dx_{||}^2 - \frac{a^2}{\tau^2} d\tau^2 + a^2 d\tilde{\Omega}_5^2, \tag{9.9}$$

where

$$d\tilde{\Omega}_5^2 = d\beta^2 + \sinh^2\beta\, d\Omega_4^2 \tag{9.10}$$

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is the metric on H^5 of 'radius' 1, and the longitudinal metric is now $dx_{||}^2 = dx_1^2 + \cdots + dx_4^2$. The metric

$$ds^2 = \frac{\tau^2}{a^2} dx_{||}^2 - \frac{a^2}{\tau^2} d\tau^2 \tag{9.11}$$

is the de Sitter metric of 'radius' a.

In the region $r^2 > t^2$, we use (9.5) with

$$H^{1/2} = \frac{a^2}{\sigma^2}\,, \tag{9.12}$$

($\sigma^2 = r^2 - t^2$) and define the coordinates σ, α by

$$r = \sigma \cosh\alpha, \qquad t = \sigma \sinh\alpha\,, \tag{9.13}$$

so that the metric (9.5) becomes the $dS_5 \times H^5$ metric

$$ds^2 = \frac{\sigma^2}{a^2} dx_{||}^2 + \frac{a^2}{\sigma^2} d\sigma^2 + a^2 d\hat{\Omega}_5^2\,, \tag{9.14}$$

where

$$d\hat{\Omega}_5^2 = -d\alpha^2 + \cosh^2\alpha\, d\Omega_4^2 \tag{9.15}$$

is the de Sitter metric of radius 1, and the metric

$$ds^2 = \frac{\sigma^2}{a^2} dx_{||}^2 + \frac{a^2}{\sigma^2} d\sigma^2 \tag{9.16}$$

is the metric on H^5 of radius a. The light-cone $r^2 = t^2$ is the boundary of H^5 at $\sigma = 0$, and is at an infinite distance from any interior point.

In either case, the limit $c = 0$ describes the geometry near the light-cone $r^2 = t^2$, while far away from the light-cone $|t^2 - r^2| >> a^2$, $H \sim 1$ and the metric approaches that of flat space. Thus the two regions of the E4-brane solution both interpolate between the 10-dimensional Minkowski metric and the $dS_5 \times H^5$ metric, but in one case the interpolation is spacelike and in the other it is timelike.

The coordinates used above only cover part of the relevant spaces. In the case $a = 0$, we have flat 10-dimensional Minkowski space and the Rindler-type coordinates $\tau, \beta, \ldots$ or $\sigma, \alpha, \ldots$ only cover either $\mathbb{R}^4 \times C_{int}$ or $\mathbb{R}^4 \times C_{ext}$, where C_{int} is the interior of the light-cone $t^2 > r^2$ in 6-dimensional Minkowski-space, and C_{ext} is the exterior of the light-cone $r^2 > t^2$. Neither the interior or the exterior region of Minkowski space is geodesically complete, as the two regions can be linked by spacelike geodesics. However, if $a \neq 0$, there is a coordinate singularity (at least) on the light-cone $r^2 = t^2$, and the global structure needs to be considered more carefully. With $c = 0$, the space is $dS_5 \times H^5$, and so non-singular.

Consider first the exterior of the light-cone $t^2 < r^2$, in which the coordinates α and the S^4 coordinates cover the whole of dS_5, and the coordinates $\sigma, x_{||}$ with $\sigma > 0$ cover the whole of H^5. In this representation, H^5 is the half-space $\sigma > 0$ in $\mathbb{R}^5$ with boundary

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given by the hyperplane $\sigma = 0$ together with a point at infinity. The light-cone $r^2 = t^2$ is then the boundary $\sigma = 0$ of H^5, which is at infinite geodesic distance from any interior point, so that now the space is geodesically complete and there are no finite length curves passing through the light-cone to the other region. The same remains true for the E4-brane with $c = 1$, so as one tries to approach the light-cone $r^2 = t^2$, the space becomes the asymptotic $dS_5 \times H^5$ geometry and the distance to the light-cone $\sigma = 0$ becomes infinite. The E4-brane solution given by (9.14) with σ real and positive (i.e. $r^2 > t^2$) is geodesically complete and non-singular and no other region is needed.

Similarly, consider the interior of the light-cone $r^2 < t^2$. In the limit $c = 0$, the coordinates β and the S^4 coordinates cover the whole of H^5 (which is half of the 2-sheeted hyperboloid), but the coordinates $\tau, x_{||}$ with $\tau > 0$ cover only half of the de Sitter space. There is a coordinate singularity at $\tau = 0$, and the region near the boundary $\tau = 0$ is best described by representing dS_5 as the hyperboloid (8.5) in the Minkowski space with metric (8.6). Then $\tau = T + Y$ (by an argument similar to that in the appendix of [26]), so $\tau = 0$ is the intersection of the de Sitter hyperboloid (8.5) with the null hyperplane $Y + T = 0$, and the region $\tau > 0$ is the half of the hyperboloid in which $T + Y > 0$ (corresponding to a 'steady-state universe' solution). Then $\tau = 0$ is a coordinate singularity, and the geometry can be continued through this to give the complete non-singular $dS_5 \times H^5$ solution. The interior of the light-cone splits into two regions, the past light-cone $t < r < 0$ and the future light-cone $0 < r < t$, and it is natural to define the proper time so that these are the regions $\tau < 0$ and $\tau > 0$, so that these correspond to the two halves of the de Sitter space, $T + Y < 0$ and $T + Y > 0$. For the E4-brane solution with $t^2 > r^2$, the region near $t^2 = r^2$ or $\tau = 0$ is described by a non-singular $dS_5 \times H^5$ geometry, and $\tau \to -\tau$ is an isometry, so that one can argue as in [50] that the space can be continued through the coordinate singularity at $\tau = 0$. Then the region τ real or $t^2 > r^2$ of the E4-brane solution is also a complete non-singular solution.

There are thus two distinct complete E4-brane solutions, corresponding to the interior or the exterior of the light-cone, giving rise to a timelike or a spacelike interpolation. In [26], N parallel D3-branes separated by distances of order ρ were considered and the zero-slope limit $\alpha' \to 0$ was taken keeping $r = \rho/\alpha'$ fixed, so that the energy of stretched strings remained finite. This decoupled the bulk and string degrees of freedom leaving a theory on the brane which is $U(N)$ $\mathcal{N} = 4$ super Yang-Mills with Higgs expectation values, which are of order r, corresponding to the brane separations. The corresponding D3-brane supergravity solution is of the form (9.1), (9.2) and has charge $q = a^2 \propto N g_s/\alpha'^2$ where g_s is the string coupling constant, which is related to the super Yang-Mills coupling constant g_{YM} by $g_s = g_{YM}^2$. Then as $\alpha' \to 0$, q becomes large and the background becomes $AdS_5 \times S^5$. The IIB string theory in the $AdS_5 \times S^5$ background is a good description if the curvature $R \sim 1/a^2$ is not too large, while if a^2 is large, the super Yang-Mills description is reliable. In the 't Hooft limit in which N

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becomes large while $g_{YM}^2 N$ is kept fixed, $g_s \sim 1/N$, so that as $N \to \infty$, we get the free string limit $g_s \to 0$, while string loop corrections correspond to $1/N$ corrections in the super Yang-Mills theory.

Similar arguments apply here, with the two E4-brane solutions with spacelike or timelike interpolations corresponding to whether the separation between the E4-branes that is kept fixed is spacelike or timelike. Recall that the scalars of the super Yang-Mills theory are in a vector representation of the $SO(5,1)$ R-symmetry, where those in the **5** of $SO(5) \subset SO(5,1)$ have kinetic terms of the right sign and correspond to brane separations in the 5 spacelike transverse dimensions, while the remaining ($U(N)$-valued) scalar is a ghost and corresponds to timelike separations of the E-branes.

Consider first the case of N parallel E4-branes of the IIB* string theory separated by distances of order ρ in one of the 5 spacelike transverse dimensions. We take the zero-slope limit $\alpha' \to 0$ keeping $\sigma = \rho/\alpha'$ fixed, so that the energy of stretched strings remains finite. This gives a decoupled theory on the brane consisting of the $U(N)$ $\mathcal{N} = 4$ Euclidean super Yang-Mills, with Higgs expectation values of order σ for the scalars corresponding to the spacelike separations. The corresponding supergravity background is the E4-brane with spacelike interpolation, arising from the outside of the light-cone with σ real and positive. We again have $q = a^2 \propto N g_s/\alpha'^2$ and $g_s = g_{YM}^2$, so that for large N, the system can be described by the IIB* string theory in $dS_5 \times H^5$ if a^2 is large and by the large N Euclidean super Yang-Mills theory when a^2 is small. In the 't Hooft limit, string loop corrections again correspond to $1/N$ corrections in the super Yang-Mills theory. In the same sense that in the anti-de Sitter correspondence, the Lorentzian gauge theory can be thought of as located at the timelike boundary of anti de Sitter space, here the Euclidean gauge theory can be thought of as located at the boundary S^4 of H^5.

For N E4-branes of the IIB* string theory separated by distances of order T in the timelike transverse dimension, we take the zero-slope limit $\alpha' \to 0$ keeping $\tau = T/\alpha'$ fixed. This gives a decoupled theory on the brane consisting of the $U(N)$ $\mathcal{N} = 4$ Euclidean super Yang-Mills, with Higgs expectation values of order τ for the scalars corresponding to the timelike separations. The corresponding supergravity background is the E4-brane with timelike interpolation, arising from the inside of the light-cone with τ real. Again for large N, the system can be described by the IIB* string theory in $dS_5 \times H^5$ if a^2 is large and by the large N Euclidean super Yang-Mills theory when a^2 is small. In this case, the Euclidean gauge theory can be thought of as being located at the past (or future) Cauchy surface $\tau = -\infty$ ($\tau = \infty$).

10. Euclideanised branes and de Sitter space

The conjectured correspondence between large N super Yang-Mills theory and IIB string theory in $AdS_5 \times S^5$ has been made more precise in [36, 47], where correlation

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functions of the super Yang-Mills theory are given by the dependence of the IIB string theory S-matrix on the asymptotic behaviour at infinity. In [36], this is formulated in terms of the Euclideanised or Wick-rotated theory; the Euclidean version of AdS_5 is the hyperboloid H^5 with boundary S^4 and correlation functions of the Euclideanised super Yang-Mills theory (with $SO(6)$ R-symmetry) on S^4 are related to the dependence of the S-matrix elements of the 'Euclideanised IIB string theory' on $H^5 \times S^5$ on the boundary conditions on S^4.

This can be motivated as follows. Consider the Dq instanton

$$ds^2 = H^{-1/2}(dx_1^2 + \cdots + dx_q^2) + H^{1/2}(dx_{q+1}^2 + \cdots + dx_{10}^2)\,, \tag{10.1}$$

where H is a harmonic function of the transverse coordinates $x_{q+1}, \ldots, x_{10}$, such as

$$H = c + \frac{q}{r^{8-q}}\,, \tag{10.2}$$

where c is a constant and r is now the radial coordinate in the transverse space defined by

$$r^2 = \sum_{i=q+1}^{10} x_i^2\,. \tag{10.3}$$

This arises from the Wick rotation $t \to it$ of the D$q-1$ brane solution (7.1).

With $c = 1$, the Dq-instanton interpolates between a flat space region where r^2 is large and a small r region where the constant term in (10.2) is negligible. For $q = 4$, the metric in this region is given by

$$ds^2 = H^{-1/2}(dx_1^2 + \cdots + dx_4^2) + H^{1/2}(dr^2 + r^2 d\Omega_5^2)\,, \tag{10.4}$$

with

$$H = \frac{a^4}{r^4}\,, \tag{10.5}$$

which can be written as

$$ds^2 = \frac{r^2}{a^2} dx_{||}^2 + \frac{a^2}{r^2} dr^2 + a^2 d\Omega_6^2\,, \tag{10.6}$$

and is the metric on $H^5 \times S^5$. Thus the D4-instanton interpolates between flat space and $H^5 \times S^5$ and in the large N and zero-slope limit, as in [26], if a is large the theory is well described by the Euclideanised string theory in $H^5 \times S^5$, and when a is small, it is well described by the large N limit of the Euclideanised super Yang-Mills; this leads to the Euclideanised version of the Maldacena duality described above.

The conjectured equivalence between the Euclidean super Yang-Mills theory and the IIB* string theory on $dS_5 \times H^5$ should similarly have a Euclideanised formulation. Yang-Mills theory in $d+1$ Lorentzian dimensions is Euclideanised by Wick rotating $t \to it$ and taking $A_0 \to -iA_0$, so that the connection 1-form remains real and the

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Euclidean action is positive. From the super Yang-Mills theory in $9+1$ dimensional space, we can obtain a super Yang-Mills theory in D-dimensional Minkowski space by reducing on a spacelike torus T^{10-D} and a super Yang-Mills theory in D-dimensional Euclidean space by reducing on a timelike torus $T^{9-D,1}$, whereas the Euclideanised theory in 10 dimensions can be reduced on T^{10-D} to give a theory in D dimensions. It is natural to take this as the Euclideanisation of both of the D-dimensional super Yang-Mills theories; for the one in D-dimensional Minkowski space, this is the usual analytic continuation, while for the Euclidean theory, this amounts to continuing the scalar ϕ in D dimensions arising from A_0 by $\phi \to -i\phi$, so that its action becomes positive and the R-symmetry changes from $SO(9-D,1)$ to $SO(10-D)$.

The Euclidean version of the de Sitter space dS_5 is the 5-sphere S^5; after analytically continuing the appropriate time coordinate, there is a coordinate singularity which can be removed by identifying the Euclidean time with period given by the inverse of the Gibbons-Hawking temperature of de Sitter space, associated with the presence of a cosmological event horizon [51]. Thus the Euclidean version of $dS_5 \times H^5$ is $S^5 \times H^5$, which is the same as the Euclidean version of $S^5 \times AdS_5$; the same space has two different Lorentzian continuations, depending on which factor in the product is continued, and the Euclidean field theory equivalence of [36] can be continued in one of two ways, to give either the de Sitter space duality or the anti-de Sitter space duality.

The Wick rotation $t \to it$ of the Eq-brane solution (7.4) gives the same Dq instanton solution (10.1), (10.2) as was obtained from analytically continuing the D$q-1$ brane, and for $q=4$ this again interpolates between flat space and $S^5 \times H^5$. (Note that the inside of the light-cone has become a point, so there is only one type of Euclideanised E-brane.) Both the Euclideanised type II and type II^* theories are expected to admit the same D-instanton solutions and similarly they should admit instantons corresponding to the Wick rotation of strings and NS 5-branes; the NS-NS sectors are the same for both the II and II* theories and so these couple to the same NS 1-branes and 5-branes. Thus the Euclideanisations of the type II and type II^* theories have the same instanton-brane spectrum and so can be taken to be the same theory in 10 Euclidean dimensions. This will be the case if the analytic continuation of the type II* theories is accompanied by an extra continuation $C_n \to -iC_n$ of the RR gauge fields so that the Euclideanised action of the matter fields becomes positive and is the same as that of the Euclideanised IIB theory; these Euclideanised actions are discussed in an appendix. This is similar to the redefinition $\phi \to -i\phi$ of the scalar ghost in the Euclidean super Yang-Mills theory above, so that the Euclidean and Lorentzian D-dimensional super Yang-Mills theories have the same Euclideanised version.

Then the Euclideanised version of the duality between IIB* string theory $dS_5 \times H^5$ and 4-dimensional Euclidean super Yang-Mills is the *same* as the Euclideanised version of the duality between IIB string theory $AdS_5 \times S^5$ and 4-dimensional Lorentzian super Yang-Mills. In both cases, it a duality between the Euclideanised IIB theory on

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$H^5 \times S^5$ and 4-dimensional Euclideanised super Yang-Mills, and this is the duality that was formulated in [36] and for which some evidence now exists. The same dual pair of Euclideanised theories has two different Lorentzian sections. For the space $H^5 \times S^5$, one either continue the H^5 to AdS_5 or the S^5 to dS_5; for the Eulideanised super Yang-Mills theory given by reducing from 10 Euclidean dimensions on $\mathbb{R}^4 \times T^6$, one can either continue one of the coordinates in $\mathbb{R}^4$ to get a theory in Minkowski space with $SO(6)$ R-symmetry, or one can continue one of the internal T^6 coordinates, to obtain a theory with $SO(5,1)$ R-symmetry in Euclidean space; for the D4-instanton, one can either continue one of the longitudinal coordinates to obtain a D3-brane or one can continue one of the transverse directions to obtain an E4-brane.

Thus a precise Euclideanised formulation of the de Sitter duality is the same as that of the anti-de Sitter duality; it is a relation between correlation functions of the Euclideanised super Yang-Mills, and the S-matrix of the Euclideanised string theory, but this can be continued back to a Lorentzian regime in two different ways. Continuing back to the E4-brane with the spacelike interpolation naturally leads to a holographical association of the bulk theory with the Euclidean gauge theory on the boundary $\sigma = 0$ of H^5. For the timelike interpolation, the natural association would appear to be with the Euclidean gauge theory on the hypersurface $\tau = 0$ of the de Sitter space.

11. Interpretation of E-branes and vacuum instability

In this section, the geometry and interpretation of the E4-brane solution will be considered further. It is is closely related to the extreme Reissner-Nordstrom solution in 4 dimensions considered in [17]. The Euclideanised Reissner-Nordstrom geometry can be continued back to Lorentzian signature in two ways, to give either the usual extreme Reissner-Nordstrom metric which interpolates between $AdS_2 \times S^2$ and flat space, or to give an E-brane type solution which interpolates between $dS_2 \times H^2$ and the flat metric. In [17], the Euclideanised solution was interpreted as an instanton, and the analytic continuation to an E-brane type solution was interpreted as an instability in which a wormhole expands at the speed of light, leading to a decay of the Minkowski-space vacuum.

The E4-brane geometry is

$$ds^2 = H^{-1/2}(dx_1^2 + \cdots + dx_4^2) + H^{1/2}(-dt^2 + dr^2 + r^2 d\Omega_4^2)\,, \tag{11.1}$$

with

$$H = 1 + \frac{a^4}{(r^2 - t^2)^2}\,. \tag{11.2}$$

The spatial slice $t = 0$ is a wormhole with an infinite throat whose cross-section is an S^4, and which interpolates between flat space (at large r) and the throat geometry $H^5 \times S^4$ (at small r). As t increases, the radius of the S^4 of fixed $r, x_1, ..., x_4$ increases

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from a at $t = 0$ to infinity at $t = r$, and the wormhole expands at the speed of light. For the complete solution with $r^2 > t^2$, the area $r^2 < t^2$ is excluded, and so as t increases, an ever increasing region of the $\mathbb{R}^5$ parameterised by r and the S^4 coordinates is excluded. Thus the wormhole 'eats up' the whole of this $\mathbb{R}^5$, and such wormholes are spontaneously produced throughout $\mathbb{R}^5$ at a certain rate per unit 5-volume. This can be interpreted as an instability of the Minkowski space vacuum of the type IIB* theory, arising from the tunneling associated with the D4-instanton.

In terms of the coordinates σ, α defined by (9.13), the solution for the region $r^2 > t^2$ is

$$ds^2 = \left(\frac{\sigma^4}{a^4+\sigma^4}\right)^{1/2} dx_{||}^2 + \left(\frac{a^4+\sigma^4}{\sigma^4}\right)^{1/2} d\sigma^2 + (a^4+\sigma^4)^{1/2} d\hat{\Omega}_5^2\,, \tag{11.3}$$

where $d\hat{\Omega}_5^2$ is the 5-dimensional de Sitter metric of radius 1. As we have seen, this interpolates between flat space and $dS_5 \times H^5$. A slice of constant σ is $\mathbb{R}^4 \times dS_5$, where the radius of the de Sitter space is $(a^4+\sigma^4)^{1/4}$ and increases from a at the boundary $\sigma = 0$ to infinity as σ becomes large. For fixed σ and α, the space is $\mathbb{R}^4 \times S^4$, and as the de Sitter time α evolves, the S^4 contracts to a minimum radius a at time $\alpha = 0$ and then expands. An observer in the space-time (9.14)sd would interpret this as a universe expanding in four spatial dimensions, and the region $t^2 > r^2$ in which σ is imaginary would not be part of his universe. To him, the idea that there is a region outside his universe that is being 'eaten up' would not be testable.

Similarly, consider the region $t^2 > r^2$. In terms of the coordinates τ, β defined in (9.8), the E4-brane metric is

$$ds^2 = \left(\frac{\tau^4}{a^4+\tau^4}\right)^{1/2} dx_{||}^2 - \left(\frac{a^4+\tau^4}{\tau^4}\right)^{1/2} d\tau^2 + (a^4+\tau^4)^{1/2} d\tilde{\Omega}_5^2\,, \tag{11.4}$$

where $d\tilde{\Omega}_5^2$ is the metric on H^5 of 'radius' 1. The space interpolates between flat space near $\tau = \infty$ or $\tau = -\infty$ and $dS_5 \times H^5$ near $\tau = 0$. A slice of constant time τ is $\mathbb{R}^4 \times H_5$, where the radius of the hyperbolic space H^5 is $(a^4+\tau^4)^{1/4}$ and so starts being infinitely large in the past at $\tau = -\infty$, decreases to radius a at $\tau = 0$ and then re-expands indefinitely. Again, an observer would see an expanding universe, and would not see anything outside his universe that was being 'eaten up' as his universe expands.

This suggests the following interpretation. For the type II* theories, the presence of ghosts leads to an instability of the flat space vacuum. There is an instability associated with each of the Dq-instantons for each q, which correspond to worm-holes expanding and eating up the transverse space, in the same way we have seen above for the D4 instanton. This is a 'tunneling to nothing', as in the decay of the Kaluza-Klein vacuum [53]. However, whereas flat space is unstable, the IIB* theory has a maximally supersymmetric $dS_5 \times H^5$ solution, with a de Sitter cosmology. There are similar four and seven dimensional de Sitter solutions of the 11-dimensional analogue of the type IIA* theory [43]. The E4-brane solution (7.4) leads to two separate geodesically

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complete solutions, each of which interpolates between the de Sitter cosmology and flat space. The interpretation of the other E-branes will be considered further in [43].

However, it is also interesting to consider the full E4-brane solution (11.1) for all values of r, t. The space-time is divided into two regions by the light-cone $r^2 = t^2$, each of which is non-singular and complete, as discussed above. As t increases (with $t > 0$), the exterior region $r^2 > t^2$ gets smaller, but now instead of a 'tunneling to nothing', there is a tunelling to the interior solution $t^2 > r^2$, which grows to fill up the transverse space, and is flat in the limit $t \to \infty$. This could be interpreted as follows: there is tachyonic brane creating a spherical 'shock-wave' in the transverse space expanding at the speed of light. This divides the space into two regions, $t^2 < r^2$ and $t^2 > r^2$, and distorts the geometry so that geodesics do not reach the boundary $r^2 = t^2$, and communication between the two regions is impossible. Far away from the light-cone in either region, the space becomes flat. As the hole inside the expanding bubble has been filled in, this would represent a shock-wave, not an instability. An observer in the exterior region $r^2 > t^2$ cannot tell whether there is nothing outside his universe, so that the Minkowski vacuum can be said to have decayed, or whether there is another region on 'the far side', separated from his observable universe by a shock-wave, in which case the Minkowski vacuum could be said to be regained once the shock-wave has passed.

12. De Sitter topological gravity and large N topological field theory

The Euclidean super Yang-Mills theory has ghosts but can be twisted to obtain a topological field theory, in which the physical states are the BRST cohomology classes. The B-model, for example, arises from twisting the $SO(4)$ Lorentz symmetry with an $SO(4)$ subgroup of the $SO(5,1)$ R-symmetry. However, the original theory has $SO(5,1)$ conformal symmetry, and the twisted theory should also be conformal. This can be made manifest by twisting the $SO(5,1)$ conformal symmetry with the $SO(5,1)$ R-symmetry, so that there is a supercharge that is a singlet under the diagonal $SO(5,1)$. This gives a BRST charge of the topological theory that is invariant under the twisted conformal group $SO(5,1)$. The B-model obtained by twisting $SO(4)$ is in fact conformally invariant under the diagonal $SO(5,1)$ of the original $SO(5,1) \times SO(5,1)$ symmetry. The A model and the half-twisted model arise from twisting the $SU(2)_L$ subgroup of the $SO(4) \sim SU(2)_L \times SU(2)_R$ Lorentz group with the two different embeddings of $SU(2)$ in the R-symmetry $SO(5,1)$.

If the Euclidean super Yang-Mills theory is dual to the type IIB* theory in $dS_5 \times H^5$, then a twisting of the super Yang-Mills theory should correspond to a twisting of the type IIB* theory, which would then be a topological string theory with a corresponding topological gravity limit. This should be particularly useful for studying the large N limit of the topological gauge theory, as the supergravity dual should give an alternative

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means of calculating correlation functions which may in some cases be much easier than the direct calculation. The supersymmetry generators of the super Yang-Mills theory correspond to the global supersymmetries of the supergravity theory, arising from the 32 Killing spinors of the de Sitter vacuum. After the twisting, one linear combination of the super Yang-Mills supersymmetry generators becomes the BRST charge of the topological field theory, and so the corresponding linear combination of the Killing supersymmetries should lead to the BRST transformation on the supergravity or superstring side. After twisting, the Killing spinor corresponding to the BRST transformations becomes a 'Killing scalar' or constant. Whereras only a special class of spaces admit Killing spinors, all admit Killing scalars, and so the superstring theory might be expected to remain topological on a more general class of background. These topological models will be discussed further elsewhere.

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A. Wick Rotations and Euclidean Path Integrals

In the analysis of section 10, use was made of the Euclideanised type II supergravity theories, and it may be useful to discuss these here. We will ignore the issues of the fermionic sector and concentrate on the bosonic actions. Recall that for D-dimensional Einstein-Maxwell theory, the Wick rotation is accompanied by the analytic continuation $A_0 \to -iA_0$ of the time component of the potential, so that the connection one-form remains real and the resulting Maxwell action is positive definite and $SO(D)$ invariant; see e.g. [54]. The Euclideanised Lagrangian is then proportional to $-R + F^2$. Then solutions (e.g. Euclideanised Reissner-Nordstrom) carrying only a magnetic charge are real, while those carrying electric charge must have an imaginary electric charge, so that A_0 is pure imaginary. If instead one were to continue the spatial part of the potential $A_i \to iA_i$ instead of the time component, the resulting theory would have a Lagrangian proportional to $-R - F^2$. with a negative definite Maxwell action. However, with this continuation, the Wick rotation of the electrically charged solutions would be real, but the magnetically charged ones would have an imaginary magnetic charge and imaginary vector potential. This is reflected in the fact that if the the vector field is dualised to a $D-3$ form gauge field $\tilde{A}$, then $F^2 = -\tilde{F}^2$ so that the sign of the kinetic term of the dual gauge field $\tilde{A}$ would be the opposite to that of A. In the Euclidean path integral, it is the action $-R + F^2$ that is used if one integrates over A (as opposed to $\tilde{A}$) as it is positive (apart from the conformal gravitational mode, which can be separately continued), while if one integrates over $\tilde{A}$, one uses $-R + \tilde{F}^2$.

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The Euclideanised bosonic part of the IIA supergravity action is (see e.g. [23])

$$I_{IIA} = \int d^{10}x\sqrt{g}\left[e^{-2\Phi}\left(-R - 4(\partial\Phi)^2 + H^2\right) + G_2^2 + G_4^2\right] + \frac{4i}{\sqrt{3}}\int G_4 \wedge G_4 \wedge B_2 + \cdots \tag{A.1}$$

This results from the Wick rotation $t \to it$ accompanied by the analytic continuation $A_{0i\ldots.j} \to -iA_{0i\ldots.j}$ for the electric components of the gauge fields B_2, C_1, C_3, so that the n-form potentials remain real and the Euclideanised action for these fields is positive. The Wick-rotation of the magnetically charged p-branes (i.e. those with $p \geq 4$) to $p+1$-instantons are real solutions of the Euclideanised theory, while the electrically charged ones (i.e. those with $p < 4$) rotate to solutions in which the electric brane charge is imaginary, so the solutions have a real metric and dilaton, but the $p+1$ form gauge field is pure imaginary. On the other hand, similar steps lead to the following action for the Euclideanised version of the IIA* theory

$$I_{IIA^*} = \int d^{10}x\sqrt{g}\left[e^{-2\Phi}\left(-R - 4(\partial\Phi)^2 + H^2\right) - G_2^2 - G_4^2\right] - \frac{4i}{\sqrt{3}}\int G_4 \wedge G_4 \wedge B_2 + \cdots \tag{A.2}$$

Then the Dn-instantons from Wick rotating En-branes with $n \leq 4$ are real, while the those from Wick rotating En-branes with $n > 4$ carry imaginary charge and couple to an imaginary RR potential. Moreover, the RR kinetic terms are negative definite (unless dualised). However, instead of analytically continuing the electric RR potentials $C_{0i..j} \to -iC_{0i..j}$, one could instead continue the spatial components $C_{i..j} \to iC_{i..j}$, and this gives the Euclideanised IIA action (A.1). If one is to integrate over the RR potentials C_1, C_3, then this is the Euclideanised action that is appropriate as the RR kinetic terms are positive, while if one were to integrate over the dual potentials $\tilde{C}_7, \tilde{C}_5$, it would be the Euclideanised action (A.2), expressed in terms of $\tilde{C}_7, \tilde{C}_5$, that would be used. This is the sense in which the IIA and the IIA* theories can be continued to the same Euclideanised action, which is (A.1) if one integrates over C_1, C_3, but would have been (A.2) if one had integrated over the dual potentials.

Similar considerations apply to the IIB theory. The Euclideanised bosonic action can be taken to be

$$I_{IIB} = \int d^{10}x\sqrt{g}\left[e^{-2\Phi}\left(-R - 4(\partial\Phi)^2 + H^2\right) + G_1^2 + G_3^2 + G_5^2\right] + \cdots, \tag{A.3}$$

but now the field equations cannot be supplemented by the constraint $G_5 = *G_5$, as in 10 Euclidean dimensions $**G_5 = -G_5$ and this would imply $G_5 = 0$, while $G_5 = i*G_5$ would require a complex C_4. This is the Euclideanised action for either the IIB or IIB* theories if one integrates over the potentials C_0, C_2, C_4 for either the IIB or the IIB* theories, but the sign of the kinetic terms would be reversed if one were

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to regard the dual potentials as fundamental. With the signs of (A.3), the D0 and D2 instantons carry imaginary charge; in particular, the D0 instanton or D instanton has an imaginary RR scalar background, but the action governing the fluctuations in C_0 is positive and indeed the action (A.3) has scalars taking values in $SL(2,\mathbb{R})/SO(2)$. However, the action obtained from this by $C_0 \to -iC_0$ would have a D-instanton with real C_0 and scalars taking values in $SL(2,\mathbb{R})/SO(1,1)$, as in [22]. This would be the appropriate action if one were to integrate over the dual potential $\tilde{C}_8$, instead of C_0 as $d\tilde{C}_8^2 = -dC_0^2$.

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